Second-order matter density perturbations and skewness in scalar-tensor modified gravity models

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We study second-order cosmological perturbations in scalar-tensor models of dark energy that satisfy local gravity constraints, including $f(R)$ gravity. We derive equations for matter fluctuations under a sub-horizon approximation and clarify conditions under which first-order perturbations in the scalar field can be neglected relative to second-order matter and velocity perturbations. We also compute the skewness of the matter density distribution and find that the difference from the ΛCDM model is only less than a few percent even if the growth rate of first-order perturbations is significantly different from that in the ΛCDM model. This shows that the skewness provides a model-independent test for the picture of gravitational instability from Gaussian initial perturbations including scalar-tensor modified gravity models.

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I. INTRODUCTION

The constantly accumulating observational data [1] continue to confirm that the Universe has entered the phase of an accelerated expansion after the matter-dominated epoch. To reveal the origin of dark energy (DE) responsible for this late-time acceleration is one of the most serious stumbling block in modern cosmology [2, 3]. The first step toward understanding the nature of DE is to find a signature whether it originates from some modification of gravity or it comes from some exotic matter with negative pressure. If gravity is modified from Einstein's General Relativity, this leaves a number of interesting experimental and observational signatures that can be tested. Especially local gravity experiments generally place tight bounds for the parameter space of modified gravity models.

So far many modified gravity DE models have been proposed– ranging from $f(R)$ gravity [4] (R is a Ricci scalar), scalar-tensor theory [5, 6] to braneworld scenarios [7]. The $f(R)$ gravity is presumably the simplest generalization to the Λ-Cold Dark Matter (ΛCDM) model ($f(R) = R - \Lambda$). Nevertheless it is generally not easy to construct viable $f(R)$ models that satisfy all stability, experimental and observational constraints while at the same time showing appreciable deviations from the ΛCDM model. In order to avoid that a scalar degree of freedom (scalaron) as well as a graviton becomes ghosts or tachyons we require the conditions $f_{,RR} > 0$ and $f_{,R} > 0$ [8]. These conditions are also needed for the stability of density perturbations [9]. For the existence of a matter-dominated epoch followed by a late-time acceleration, the models need to be close to the ΛCDM model ($m \equiv Rf_{,RR}/f_{,R} \approx +0$) in the region $R \gg R_0$ (R_0 is the present cosmological Ricci scalar) [10]. Moreover the mass of the scalaron field in the region $R \gg R_0$ is sufficiently heavy for the compatibility with local gravity experiments [11, 12, 13, 14]. Finally, for the presence of a stable de Sitter fixed point at $r \equiv -Rf_{,R}/f = -2$, we require that $0 \le m(r = -2) \le 1$ [10, 15]. The models proposed by Hu and Sawicki [16] and Starobinsky [8] satisfy all these requirements. They take the asymptotic form, $f(R) \simeq R - \mu R_c[1 - (R/R_c)^{-2n}]$ ($\mu > 0, R_c > 0, n > 0$), in the region $R \gg R_c$ (R_c is roughly the same order as R_0). See Refs. [17, 18, 19, 20, 21] for other viable $f(R)$ models.

The main reason why viable $f(R)$ models are so restrictive is that the strength of a coupling Q between dark energy and non-relativistic matter (such as dark matter) is large in the Einstein frame ($Q = -1/\sqrt{6}$) [22]. In the region of high-density where local gravity experiments are carried out, the scalaron field ϕ needs to be almost frozen [11, 12] with a large mass through a chameleon mechanism [23] to avoid that the field mediates a long ranged fifth force.

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Cosmologically this means that the field does not approach a kinematically driven ϕ matter-dominated era ("ϕMDE" [24]) in which the evolution of scale factor is non-standard ($a \propto t^{1/2}$ [22]). The deviation from the ΛCDM model becomes important as the mass of the scalaron gets smaller so that the field begins to evolve slowly along its potential. In other words the effect of modified gravity manifests itself from the late-time matter era to the accelerated epoch [8, 16]. This leaves a number of interesting observational signatures for the equation of state of DE [18, 20], matter power spectra [8, 9, 20] and convergence spectra in weak lensing [25, 26].

One can generalize the analysis in $f(R)$ gravity to the theories that have arbitrary constant couplings Q [27]. In fact this is equivalent to Brans-Dicke theory [28] with a scalar-field potential $V(\phi)$. By designing the potential so that the field mass is sufficiently heavy in the region of high density, it is possible to satisfy both local gravity and cosmological constraints even when $|Q|$ is of the order of unity [27]. The representative potential of this type is given by $V(\phi) = V_0[1 - C(1 - e^{-2Q\phi})^p]$ ($V_0 > 0, C > 0, 0 < p < 1$), which covers the $f(R)$ models of Hu and Sawicki [16] and Starobinsky [8] as special cases. Especially when $|Q|$ is of the order of unity, these models lead to the large growth of matter density perturbations ($\delta \propto t^{(\sqrt{25+48Q^2}-1)/6}$) at a late epoch of the matter era compared to the standard growth ($\delta \propto t^{2/3}$) at an early epoch. This gives rise to a significant change of the spectral index of the matter power spectrum relative to that in the ΛCDM model [8, 20]. Moreover it was recently shown that the convergence power spectrum in weak lensing observations is subject to a large modification by the non-standard evolution of matter perturbations [25].

In this paper we shall study another test of modified gravity DE models mentioned above by evaluating a normalized skewness, $S_3 = \langle\delta^3\rangle/\langle\delta^2\rangle^2$, of matter perturbations. The skewness provides a good test for the picture of gravitational instability from Gaussian initial conditions [29]. If large-scale structure grows via gravitational instability from Gaussian initial perturbations, the skewness in a Universe dominated by a pressureless matter is known to be $S_3 = 34/7$ in General Relativity [30]. Even when cosmological constant is present at late times, the skewness depends weakly on the expansion history of the Universe (less than a few percent) [31, 32, 33]. This situation hardly changes in open/closed universes [34] and Dvali-Gabadadze-Poratti braneworld models [35]. One can see some difference for the models that are significantly different from Einstein gravity–such as Cardassian cosmologies [35, 36], modified gravity models that respect Birkhoff's law [37]. In the context of dark energy coupled with dark matter, it was shown in Ref. [38] that the skewness can be a probe of the violation of equivalence principle between dark matter and (uncoupled) baryons.

In Brans-Dicke theory with cosmological constant Λ the skewness has been calculated in Ref. [39] under the condition that the Brans-Dicke field is massless. In this case the evolution of scale factor during the matter-dominated epoch is given by $a(t) \propto t^{(2\omega_{\rm BD}+2)/(3\omega_{\rm BD}+4)}$ [39], where t is a cosmic time and $\omega_{\rm BD}$ is a Brans-Dicke parameter. If the field is massless, the Brans-Dicke parameter is constrained to be $\omega_{\rm BD} > 40000$ [40] from solar-system experiments. This shows that the evolution of the scale factor in the matter era is very close to the standard one: $a(t) \propto t^{2/3}$. We note that an effective gravitational "constant" that appears as a coefficient of matter density perturbations is also subject to change in Brans-Dicke theory. However it was found that the skewness in such a case is given by $S_3 = (34\omega_{\rm BD} + 56)/(7\omega_{\rm BD} + 12)$ [39] during the matter era, which is very close to the standard one ($S_3 = 34/7$) under the condition $\omega_{\rm BD} > 40000$.

The $f(R)$ gravity corresponds to theory with the Brans-Dicke parameter $\omega_{\rm BD} = 0$ [41]. Even in this situation, if the scalaron field has a potential whose mass is sufficiently large in the region of high density, the $f(R)$ models can pass local gravity constraints as in the models proposed in Refs. [8, 16]. In such cases, compared to Brans-Dicke theory with a massless field, it is expected that the skewness may show significant deviations from that in General Relativity. Since the evolution of scale factor and matter perturbations is different from that in the massless case, we can not employ the result of the skewness presented above.

In this paper we study second-order perturbations and the skewness for Brans-Dicke theory in the presence of a potential $V(\phi)$. This is equivalent to the scalar-field action given in Eq. (2) by identifying the coupling Q with the Brans-Dicke parameter $\omega_{\rm BD}$ via the relation $1/(2Q^2) = 3 + 2\omega_{\rm BD}$. In the massless case the solar-system constraint, $\omega_{\rm BD} > 40000$, gives the bound $|Q| \lesssim 10^{-3}$, but it is difficult to find some deviations from General Relativity in such a situation. Our interest is the case in which the coupling Q is of the order of $0.1 \lesssim |Q| \lesssim 1$ with the field potential that has a sufficiently large mass in the high-density region. This analysis includes viable $f(R)$ models [8, 16] recently proposed in the literature. We would like to investigate how much extent the skewness differs from that in the ΛCDM model. We also derive conditions under which the contribution coming from first-order field perturbations can be neglected relative to second-order matter and velocity perturbations by starting from fully relativistic second-order perturbation equations.

II. MODIFIED GRAVITY MODELS

The action for Brans-Dicke theory [28] in the presence of a potential V is given by

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}\chi R - \frac{\omega_{\rm BD}}{2\chi}(\nabla\chi)^2 - V(\chi)\right] + S_m(g_{\mu\nu}, \Psi_m)\,, \tag{1}$$

where χ is a scalar field coupled to a Ricci scalar R, $\omega_{\rm BD}$ is a so-called Brans-Dicke parameter and S_m is a matter action that depends on the metric $g_{\mu\nu}$ and matter fields Ψ_m. We shall use the unit $8\pi G = 1$, but we restore the bare gravitational constant G when it is required.

The action (1) is equivalent to the following scalar-tensor action with the correspondence $\chi = e^{-2Q\phi}$:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}F(\phi)R - \frac{1}{2}\omega(\phi)(\nabla\phi)^2 - V(\phi)\right] + S_m(g_{\mu\nu}, \Psi_m)\,, \tag{2}$$

where

$$F(\phi) = e^{-2Q\phi}\,, \quad \omega(\phi) = (1-6Q^2)F(\phi)\,. \tag{3}$$

As we already mentioned, the constant Q is related with $\omega_{\rm BD}$ via the relation $1/(2Q^2) = 3+2\omega_{\rm BD}$. In the limit $Q \to 0$ (i.e., $\omega_{\rm BD} \to \infty$), the action (2) reduces to the one for a minimally coupled scalar field ϕ with a potential $V(\phi)$. The $f(R)$ gravity corresponds to the coupling $Q = -1\sqrt{6}$, i.e., $\omega_{\rm BD} = 0$.

In the absence of the potential $V(\phi)$ the coupling Q is constrained to be $|Q| \lesssim 10^{-3}$ from solar-system tests. We are interested in the case where the presence of the potential can make the models be consistent with local gravity constraints (LGC) even for $|Q| = \mathcal{O}(1)$. The representative potential of this type is given by [27]

$$V(\phi) = V_0\left[1 - C(1-e^{-2Q\phi})^p\right] \qquad (V_0 > 0, C > 0, 0 < p < 1)\,, \tag{4}$$

where V_0 is of the order of the present cosmological Ricci scalar R_0 in order to be responsible for the acceleration of the Universe today. Note that the $f(R)$ models proposed by Hu and Sawicki [16] and Starobinsky [8] take the form $f(R) = R - \mu R_c[1-(R/R_c)^{-2n}]$ $(\mu > 0, R_c > 0, n > 0)$ in the region $R \gg R_c$. These $f(R)$ models are covered in the action (2) with (4) by identifying the field potential to be $V = (RF-f)/2$ with $F = \partial f/\partial R = e^{2\phi/\sqrt{6}}$.

The background cosmological dynamics and LGC for the potential (4) have been discussed in details in Ref. [27]. We review how the matter-dominated era is followed by the stage of a late-time acceleration. This is important when we discuss the evolution of matter density perturbations in Sec. IV. As a matter source we take into account a non-relativistic matter with an energy density ρ_m. In the flat Friedmann-Lemaitre-Robertson-Walker (FLRW) metric with scale factor $a(t)$, where t is cosmic time, the evolution equations for the action (2) are

$$3FH^2 = \frac{1}{2}\omega\dot{\phi}^2 + V - 3H\dot{F} + \rho_m\,, \tag{5}$$

$$2F\dot{H} = -\omega\dot{\phi}^2 - \ddot{F} + H\dot{F} - \rho_m\,, \tag{6}$$

$$\omega\left(\ddot{\phi} + 3H\dot{\phi} + \frac{\dot{F}}{2F}\dot{\phi}\right) + V_{,\phi} - \frac{1}{2}F_{,\phi}R = 0\,, \tag{7}$$

$$\dot{\rho}_m + 3H\rho_m = 0\,, \tag{8}$$

where $H \equiv \dot{a}/a$ is the Hubble parameter and a dot represents a derivative with respect to t. Note that the Ricci scalar is given by $R = 6(2H^2 + \dot{H})$.

We introduce the following dimensionless quantities:

$$x_1 \equiv \frac{\dot{\phi}}{\sqrt{6}H}\,, \quad x_2 \equiv \frac{1}{H}\sqrt{\frac{V}{3F}}\,, \tag{9}$$

and

$$\Omega_m \equiv \frac{\rho_m}{3FH^2} = 1 - (1-6Q^2)x_1^2 - x_2^2 - 2\sqrt{6}Qx_1\,, \tag{10}$$

where we used Eq. (5). We then obtain

$$\frac{\mathrm{d}x_1}{\mathrm{d}N} = \frac{\sqrt{6}}{2}(\lambda x_2^2 - \sqrt{6}x_1) + \frac{\sqrt{6}Q}{2}\left[(5-6Q^2)x_1^2 + 2\sqrt{6}Qx_1 - 3x_2^2 - 1\right] - x_1\frac{\dot{H}}{H^2}\,, \tag{11}$$

$$\frac{\mathrm{d}x_2}{\mathrm{d}N} = \frac{\sqrt{6}}{2}(2Q-\lambda)x_1x_2 - x_2\frac{\dot{H}}{H^2}\,, \tag{12}$$

where $N \equiv \ln(a)$ and $\lambda = -V_{,\phi}/V$. The effective equation of state is defined by

$$w_{\rm eff} = -1 - \frac{2}{3}\frac{\dot{H}}{H^2}\,, \tag{13}$$

where

$$\frac{\dot{H}}{H^2} = -\frac{1-6Q^2}{2}\left[3+3x_1^2-3x_2^2-6Q^2x_1^2+2\sqrt{6}Qx_1\right]+3Q(\lambda x_2^2-4Q)\,. \tag{14}$$

When λ is a constant (i.e., $V(\phi)=V_0e^{-\lambda\phi}$), the fixed points of the system can be derived by setting $\mathrm{d}x_1/\mathrm{d}N = \mathrm{d}x_2/\mathrm{d}N = 0$. Even if λ changes with time, as it is the case for the potential (4), the fixed points can be regarded as instantaneous ones. The following points can play the role of the matter-dominated epoch:

- (M1) ϕ matter-dominated era

$$(x_1,x_2) = \left(\frac{\sqrt{6}Q}{3(2Q^2-1)}, 0\right)\,, \quad \Omega_m = \frac{3-2Q^2}{3(1-2Q^2)^2}\,, \quad w_{\rm eff} = \frac{4Q^2}{3(1-2Q^2)}\,. \tag{15}$$

- (M2) "Instantaneous" scaling solution

$$(x_1,x_2) = \left(\frac{\sqrt{6}}{2\lambda}, \left[\frac{3+2Q\lambda-6Q^2}{2\lambda^2}\right]^{1/2}\right)\,, \quad \Omega_m = 1-\frac{3-12Q^2+7Q\lambda}{\lambda^2}\,, \quad w_{\rm eff} = -\frac{2Q}{\lambda}\,. \tag{16}$$

In order to realize the matter era ($\Omega_m \simeq 1$ and $w_{\rm eff} \simeq 0$) by the point (M1), we require the condition $Q^2 \ll 1$. This point was used in the coupled quintessence scenario [24] (in the Einstein frame) where the coupling is constrained to be $|Q| \lesssim 0.1$ from Cosmic Microwave Background anisotropies. In $f(R)$ gravity ($Q=-1/\sqrt{6}$) we have $\Omega_m = 2$ and $w_{\rm eff} = 1/3$ (i.e., $a \propto t^{1/2}$ [22]), which means that the point (M1) can not be responsible for the matter era for $|Q|$ of the order of unity.

The matter era can be realized by the point (M2) for $|\lambda| \gg |Q| = \mathcal{O}(1)$. The parameter λ for the potential (4) is given by

$$\lambda = \frac{2CpQe^{-2Q\phi}(1-e^{-2Q\phi})^{p-1}}{1-C(1-e^{-2Q\phi})^p}\,, \tag{17}$$

which is much larger than 1 for $|Q\phi| \ll 1$ (provided that C and p are not very much smaller than 1). Since $R \simeq \rho_m/F$ during the deep matter-dominated epoch, the field ϕ is stuck at the instantaneous minima characterized by the condition $V_{,\phi}(\phi_m) + Q\rho_m \simeq 0$ [see Eq. (7)]. For the potential (4) this translates into

$$\phi_m \simeq \frac{1}{2Q}\left(\frac{2V_0pC}{\rho_m}\right)^{\frac{1}{1-p}}\,, \tag{18}$$

which means that $|Q\phi_m| \ll 1$ and hence $|\lambda| \gg 1$ during the deep matter era ($\rho_m \gg V_0$). When $|Q| = \mathcal{O}(1)$, the matter era is realized by the point (M2) instead of (M1).

For the dynamical system given by Eqs. (11) and (12) there exist the following fixed points that lead to the late-time acceleration:

- (A1) Scalar-field dominated point

$$(x_1,x_2) = \left(\frac{\sqrt{6}(4Q-\lambda)}{6(4Q^2-Q\lambda-1)}, \left[\frac{6-\lambda^2+8Q\lambda-16Q^2}{6(4Q^2-Q\lambda-1)^2}\right]^{1/2}\right)\,, \quad \Omega_m = 0\,, \quad w_{\rm eff} = -\frac{20Q^2-9Q\lambda-3+\lambda^2}{3(4Q^2-Q\lambda-1)}. \tag{19}$$

- (A2) de Sitter point (present for $\lambda = 4Q$)

$$(x_1,x_2) = (0,1)\,, \quad \Omega_m = 0\,, \quad w_{\rm eff} = -1\,. \tag{20}$$

The de Sitter point (A2) appears only in the presence of the coupling Q (characterized by the condition $V_{,\phi}+QFR=0$ in Eq. (7), i.e., $\lambda = 4Q$). This can be regarded as the special case of the accelerated point (A1). For the potential (4) the parameter $|\lambda|$ is much larger than $|Q|$ during the matter era, but it gradually becomes the same order as $|Q|$ as the system enters the accelerated epoch. It was shown in Ref. [27] that the de Sitter point (A2) is stable for $\mathrm{d}\lambda/\mathrm{d}\phi < 0$. As long as $|\lambda|$ continues to decrease with the growth of $|\phi|$, the solutions are finally trapped at the stable de Sitter point (A2). If the stability condition, $\mathrm{d}\lambda/\mathrm{d}\phi < 0$, is not satisfied, the solutions approach another accelerated point (A1).

In the following we are mainly interested in the case where the "instantaneous" matter point (M2) is followed by the de Sitter point (A2). During most stages of cosmic expansion history the field ϕ is trapped at instantaneous minima of an effective potential induced by the matter coupling. This means that the condition, $\dot{\phi}^2 \ll H^2$, is well satisfied.

The mass squared of the field ϕ for the potential (4) is given by

$$M^2 \equiv V_{,\phi\phi} = 4V_0CpQ^2e^{-2Q\phi}(1-e^{-2Q\phi})^{p-2}(1-pe^{-2Q\phi})\,, \tag{21}$$

which is much larger than R_0 ($\sim V_0$) in the region $R \gg R_0$. In this situation it is possible to satisfy local gravity constraints in the region of high density [11, 12, 16] through a chameleon mechanism [23]. Since the field is massive inside a spherical symmetric body with radius r_c, only the surface part of its mass distribution contributes to the field profile outside the body. The effective coupling $Q_{\rm eff}$ between the field and the pressureless matter is suppressed by a thin-shell parameter $\Delta r_c/r_c$ relative to the bare coupling Q. For the potential (4) it was shown in Ref. [12] that constraints coming from solar system tests as well as the violation of equivalence principle give the bounds $p > 1-5/(9.6-\log_{10}|Q|)$ and $p > 1-5/(13.8-\log_{10}|Q|)$, respectively. In $f(R)$ gravity these constraints correspond to $p > 0.50$ and $p > 0.65$, respectively.

Substituting the field value (18) for Eq. (21), we find that the mass squared during the matter era is given by

$$M^2 \simeq \left(\frac{3^{2-p}}{2^p pC}\right)^{\frac{1}{1-p}}(1-p)Q^2\left(\frac{H^2}{V_0}\right)^{\frac{1}{1-p}}H^2\,, \tag{22}$$

where we used $3H^2 \simeq \rho_m$. We then find that the inequality, $M^2 \gg H^2$, holds for the values of p, C, Q not very much smaller than unity. In the next section we shall use this property when we derive the equation for matter perturbations approximately.

III. SECOND-ORDER COSMOLOGICAL PERTURBATIONS

In this section we consider second-order cosmological perturbations for the action (2) and derive the equation for matter perturbations approximately.

A. Perturbation equations

Let us start with a perturbed metric including scalar metric perturbations α, β, φ and γ about the flat FLRW background [42]:

$$\mathrm{d}s^2 = -(1+2\alpha)\mathrm{d}t^2 - 2a\beta_{,i}\mathrm{d}t\mathrm{d}x^i + a(t)^2\left[(1+2\varphi)\delta_{ij} + 2\gamma_{|ij}\right]\mathrm{d}x^i\mathrm{d}x^j\,. \tag{23}$$

At the second-order the scalar variables are written as

$$\alpha \equiv \alpha^{(1)} + \alpha^{(2)}\,,\quad \beta \equiv \beta^{(1)} + \beta^{(2)}\,,\quad \varphi \equiv \varphi^{(1)} + \varphi^{(2)}\,,\quad \gamma \equiv \gamma^{(1)} + \gamma^{(2)}\,, \tag{24}$$

where the subscripts represent the orders of perturbations. We introduce the following quantities:

$$\chi \equiv a(\beta + a\dot{\gamma})\,,\quad \kappa \equiv \delta K\,, \tag{25}$$

where δK is the perturbation of an extrinsic curvature K.

We decompose the scalar field ϕ and the quantity F into background and perturbed parts:

$$\phi = \phi_0(t) + \delta\phi(t, \boldsymbol{x})\,,\quad F = F_0(t) + \delta F(t, \boldsymbol{x})\,, \tag{26}$$

where $\delta\phi$ and δF depend on t and a position vector $\boldsymbol{x}$. In the following we omit the subscript "0" from background quantities. The components of the energy momentum tensor of a pressureless matter can be decomposed as

$$T_0^0 = -(\rho_m + \delta\rho_m)\,, \quad T_i^0 = -\rho_m v_{,i} \equiv q_{,i}\,, \tag{27}$$

where v is a rotational-free velocity potential. At the second-order, the perturbed quantities can be explicitly written as

$$\delta\phi \equiv \delta\phi^{(1)} + \delta\phi^{(2)}\,, \quad \delta F \equiv \delta F^{(1)} + \delta F^{(2)}\,, \quad \delta\rho_m \equiv \delta\rho_m^{(1)} + \delta\rho_m^{(2)}\,, \quad v \equiv v^{(1)} + v^{(2)}\,. \tag{28}$$

The perturbation equations for the action (2), up to the second-order, have been derived in Ref. [43] (see also Refs. [44]). They are given by

$$\kappa - 3H\alpha + 3\dot{\varphi} + \frac{\Delta}{a^2}\chi = -\alpha\left(\frac{9}{2}H\alpha - \frac{1}{a}\beta^{,i}{}_{|i}\right) + \frac{3}{2}H\beta^{,i}\beta_{,i}\,, \tag{29}$$

$$4\pi G\delta\rho_{\rm eff} + H\kappa + \frac{\Delta}{a^2}\varphi = \frac{1}{6}\kappa^2 - \frac{1}{4a^2}\beta_{(,i|j)}\beta^{,i|j} + \frac{1}{12a^2}(\beta^{,i}{}_{|i})^2\,, \tag{30}$$

$$\kappa + \frac{\Delta}{a^2}\chi - 12\pi G\rho a v - \frac{3}{2F}\left(\omega\dot{\phi}\delta\phi + \delta\dot{F} - H\delta F - \dot{F}\alpha\right)$$
$$= \Delta^{-1}\nabla^i\left[-\alpha(\kappa_{,i} + 12\pi G a q_{,i}) + \frac{3}{4a}\alpha_{,j}(\beta^{,j}{}_{|i} + \beta_{,i}{}^{|j}) - \frac{1}{2a}\alpha_{,i}\beta^{,j}{}_{|j}\right]\,, \tag{31}$$

$$\dot{\kappa} + 2H\kappa - 4\pi G(\delta\rho_{\rm eff} + 3\delta P_{\rm eff}) + \left(3\dot{H} + \frac{\Delta}{a^2}\right)\alpha$$
$$= \alpha\dot{\kappa} - \frac{1}{a}\kappa_{,i}\beta^{,i} + \frac{1}{3}\kappa^2 + \frac{3}{2}\dot{H}(\alpha^2 - \beta^{,i}\beta_{,i}) + \frac{1}{a^2}\left(2\alpha\alpha^{|i}{}_i + \alpha_{,i}\alpha^{,i} - \beta^{,j}\beta_{,j}{}^{|i}{}_i - \beta^{,j|i}\beta_{,j|i}\right) + \frac{1}{a^2}\beta_{(,i|j)}\beta^{,i|j} - \frac{1}{3a^2}(\beta^{,i}{}_{|i})^2, \tag{32}$$

$$\delta\dot{\rho}_m + 3H\delta\rho_m - \rho_m\left(\kappa - 3H\alpha + \frac{1}{a}\Delta v\right)$$
$$= -\frac{1}{a}\delta\rho_{m,i}\beta^{,i} + \delta\rho_m(\kappa - 3H\alpha) + \rho_m\left[\alpha\kappa + \frac{3}{2}H(\alpha^2 - \beta^{,i}\beta_{,i})\right] - \frac{1}{a}\left(\alpha q^i{}_{|i} + 2q^i\alpha_{,i}\right)\,, \tag{33}$$

$$\dot{v} + Hv - \frac{1}{a}\alpha = -\frac{1}{\rho_m}\Delta^{-1}\nabla^i\left[q_{,i}(\kappa - 3H\alpha) + \frac{1}{a}\left\{-q_{,i|j}\beta^{,j} - q_{,j}\beta^{,j}{}_{|i} - \delta\rho_m\alpha_{,i} + \rho_m(\alpha\alpha_{,i} - \beta^{,j}\beta_{,j|i})\right\}\right]\,, \tag{34}$$

where

$$\delta\rho_{\rm eff} = \frac{1}{8\pi GF}\left[\delta\rho_m + \omega(\dot{\phi}\delta\dot{\phi} - \alpha\dot{\phi}^2) + \frac{1}{2}\omega_{,\phi}\delta\phi\dot{\phi}^2 - \frac{1}{2}(F_{,\phi}R - 2V_{,\phi})\delta\phi - 3H\delta\dot{F} + \left(\frac{1}{2}R + \frac{\Delta}{a^2}\right)\delta F\right.$$
$$\left. + \left(6H\alpha - \frac{\Delta}{a^2}\chi - 3\dot{\varphi}\right)\dot{F} - \frac{\delta F}{F}\left(\rho_m + \frac{1}{2}\omega\dot{\phi}^2 + V - 3H\dot{F}\right)\right]\,, \tag{35}$$

$$\delta P_{\rm eff} = \frac{1}{8\pi GF}\left[\omega(\dot{\phi}\delta\dot{\phi} - \alpha\dot{\phi}^2) + \frac{1}{2}\omega_{,\phi}\delta\phi\dot{\phi}^2 + \frac{1}{2}(F_{,\phi}R - 2V_{,\phi})\delta\phi + \delta\ddot{F} + 2H\delta\dot{F} - \left(\frac{1}{2}R + \frac{2}{3}\frac{\Delta}{a^2}\right)\delta F\right.$$
$$\left. - 2\alpha\ddot{F} - \left(\dot{\alpha} + 4H\alpha - \frac{2}{3}\frac{\Delta}{a^2}\chi - 2\dot{\varphi}\right)\dot{F} - \frac{\delta F}{F}\left(\frac{1}{2}\omega\dot{\phi}^2 - V + \ddot{F} + 2H\dot{F}\right)\right]\,. \tag{36}$$

The equations for the perturbations $\delta\phi$ and δF are

$$\delta\ddot{\phi} + \left(3H + \frac{\omega_{,\phi}}{\omega}\dot{\phi}\right)\delta\dot{\phi} + \left[-\frac{\Delta}{a^2} + \left(\frac{\omega_{,\phi}}{\omega}\right)_{,\phi}\frac{\dot{\phi}^2}{2} + \left(\frac{2V_{,\phi} - F_{,\phi}R}{2\omega}\right)_{,\phi}\right]\delta\phi - \dot{\phi}\dot{\alpha} - \left(2\ddot{\phi} + 3H\dot{\phi} + \frac{\omega_{,\phi}}{\omega}\dot{\phi}^2\right)\alpha$$
$$-\dot{\phi}\kappa - \frac{1}{2\omega}F_{,\phi}\delta R = N_{\delta\phi}\,, \tag{37}$$

$$\delta\ddot{F} + 3H\delta\dot{F} + \left(-\frac{\Delta}{a^2} - \frac{R}{3}\right)\delta F + \frac{2}{3}\omega\dot{\phi}\delta\dot{\phi} + \frac{1}{3}(\omega_{,\phi}\dot{\phi}^2 + 2F_{,\phi}R - 4V_{,\phi})\delta\phi - \frac{1}{3}\delta\rho_m - \dot{F}(\kappa + \dot{\alpha})$$
$$-\left(\frac{2}{3}\omega\dot{\phi}^2 + 2\ddot{F} + 3H\dot{F}\right)\alpha + \frac{1}{3}F\delta R = N_{\delta F}\,, \tag{38}$$

where $N_{\delta\phi}$ and $N_{\delta F}$ are second-order terms whose explicit expressions are given in Ref. [43].

At the first-order the quantity, $\delta\rho^{(1)} \equiv \delta\rho_m^{(1)} - \dot{\rho}_m a v^{(1)}$, is known to be gauge-invariant [44]. In order to construct gauge-invariant variables at the second-order, we introduce the following quantities

$$\delta\rho \equiv \delta\rho_m - \dot{\rho}_m a v + \delta\rho^{(q)}\,, \tag{39}$$

$$v_\chi \equiv v - \frac{1}{a}\chi + v_\chi^{(q)}\,, \tag{40}$$

where $\delta\rho^{(q)}$ and $v_\chi^{(q)}$ are quadratic combinations of first-order terms. By defining $\delta_m \equiv \delta\rho_m/\rho_m$, it was shown in Ref. [43] that the following quantity is gauge-invariant at the second-order:

$$\delta \equiv \delta_m + 3aHv - \frac{\delta\dot{\rho}_v}{\rho_m} av + \frac{3}{2}\rho_m \dot{H} a^2 v^2 - v^{,i} v_{,i} - 3\rho_m a H \Delta^{-1}\nabla^i \left(\frac{\delta\rho_v}{\rho_m} v_{,i}\right)\,, \tag{41}$$

where $\delta\rho_v \equiv \delta\rho_m - \dot{\rho}_m a v$. Note that the quantity v_χ can be also made gauge-invariant [43].

B. Approximate second-order equations

If we take the temporal comoving gauge ($v = 0$), we have $\delta = \delta_m$ and $q_{,i} = 0$ [see Eqs. (27) and (41)]. Taking $\gamma = 0$ for the spatial gauge condition, it follows that $\beta = \chi/a$ from Eq. (25). From Eq. (34) we obtain

$$\alpha = -\frac{1}{2}\beta_{,i}\beta^{,i}\,, \tag{42}$$

which means that α is a second-order quantity. Up to the second-order, Eqs. (31) and (33) are written as

$$\kappa = -\frac{\Delta}{a^2}\chi + \frac{3}{2F}(\omega\dot{\phi}\delta\phi + \delta\dot{F} - H\delta F - \dot{F}\alpha)\,, \tag{43}$$

$$\dot{\delta} - \kappa = \kappa\delta - \frac{1}{a}\delta_{,i}\beta^{,i}\,. \tag{44}$$

In order to evaluate the terms on the r.h.s. of Eq. (44), it is sufficient to consider Eqs. (40) and (43) at the first order with the gauge $v = 0$. We then have $\chi^{(1)} = -a v_\chi$, $\beta^{(1)} = -v_\chi$ and

$$\kappa^{(1)} = \frac{\Delta v_\chi}{a} + \frac{3}{2F}(\omega\dot{\phi}\delta\phi + \delta\dot{F} - H\delta F)\,, \tag{45}$$

where we omitted the order of the subscript from the r.h.s. of these equations. Hence Eq. (44) can be read as

$$\dot{\delta} - \kappa = \frac{1}{a}\nabla\cdot(\delta\nabla v_\chi) + \frac{3}{2F}(\omega\dot{\phi}\delta\phi + \delta\dot{F} - H\delta F)\delta\,. \tag{46}$$

In the following we consider a situation in which the scalar-field dependent terms on the r.h.s. of Eq. (45) is neglected relative to the term $\Delta v_\chi/a$. In this case Eq. (46) yields

$$\dot{\delta} - \kappa \simeq \frac{1}{a}\nabla\cdot(\delta\nabla v_\chi)\,. \tag{47}$$

Later we shall confirm the validity of this approximation.

From Eq. (32) with Eqs. (29), (35) and (36) we obtain

$$\begin{aligned}
&\dot{\kappa} + \left(2H + \frac{\dot{F}}{2F}\right)\kappa - \frac{1}{2F}\left[\delta\rho + 4\omega\dot{\phi}\delta\dot{\phi} + (2\omega_{,\phi}\dot{\phi}^2 + F_{,\phi}R - 2V_{,\phi})\delta\phi + 3\delta\ddot{F} + 3H\delta\dot{F} - \left(6H^2 + \frac{\Delta}{a^2}\right)\delta F\right] \\
&= \frac{N_0}{2}\frac{\dot{F}}{F} + N_3 - \frac{3\dot{F}}{2F}\dot{\alpha} - \left[3\dot{H} + \frac{1}{2F}(6\ddot{F} + 6H\dot{F} + 4\omega\dot{\phi}^2) + \frac{\Delta}{a^2}\right]\alpha\,,
\end{aligned} \tag{48}$$

where N_0 and N_3 correspond to the second-order terms on the r.h.s. of Eqs. (29) and (32), respectively. Following Refs. [3, 6, 27, 45] we employ the sub-horizon approximation under which the terms containing κ, $\delta\rho$, $\Delta\,\delta F/a^2$ and

$\Delta\alpha/a^2$ are picked up in Eq. (48). Note that $|\dot{F}/HF| \ll 1$ under the condition $|\dot{\phi}| \ll H$. Apart from the term $\Delta\alpha/a^2$, the terms on the r.h.s. of Eq. (48) are of the order of $H^2\alpha$ or smaller. We then have

$$\dot{\kappa} + 2H\kappa - \frac{1}{2F}\left(\delta\rho - \frac{\Delta}{a^2}\delta F\right) \simeq \frac{1}{a^2}\left[(\nabla v_\chi)\cdot(\nabla v_\chi)^{,i}\right]_{,i} \,. \tag{49}$$

Of course this approximation is justified when the second-order term on the r.h.s. of Eq. (49) is larger than the first-order (field-dependent) terms on the l.h.s. of Eq. (48) we have neglected. Later we shall derive conditions under which this approximation is valid.

Let us estimate the field perturbation $\delta\phi$ as well as δF. As we explained in the previous section, the field mass M defined in Eq. (21) is much larger than H. Using the approximation in which the terms containing M^2, $\Delta\,\delta\phi/a^2$, $\Delta\,\delta F/a^2$, $\delta\rho$ and δR are dominant contributions to Eqs. (37) and (38), we obtain

$$\left(-\frac{\Delta}{a^2} + \frac{M^2}{\omega}\right)\delta\phi - \frac{\dot{\phi}}{a}\Delta v_\chi - \frac{1}{2\omega}F_{,\phi}\delta R \simeq 0\,, \tag{50}$$

$$-\frac{\Delta}{a^2}\delta F - \frac{1}{3}\delta\rho - \frac{\dot{F}}{a}\Delta v_\chi + \frac{1}{3}F\delta R \simeq 0\,. \tag{51}$$

This approximation is accurate as long as an oscillating mode of the field perturbation does not dominate over the matter-induced mode [8, 27]. Note that we have neglected second-order terms on the r.h.s. of Eqs. (37) and (38). Since the field is nearly frozen at the instantaneous minimum given in Eq. (18), the dominant second-order term corresponds to $V_{,\phi\phi\phi}\delta\phi^2$. This term gives rise to only a tiny correction to the growth rate of perturbations. Moreover it can be neglected relative to the second-order term on the r.h.s. of Eq. (49). See Appendix for the detailed estimation of such a second-order term.

On combining Eqs. (50) and (51), we find

$$\left(\frac{M^2}{F} - \frac{\Delta}{a^2}\right)\delta F = 2Q^2\delta\rho + \frac{\dot{F}}{a}\Delta v_\chi\,. \tag{52}$$

Note that $\delta\rho = \rho_m\delta \simeq 3FH^2\delta$ during the matter era. At the first-order we also have the following relation from Eq. (47):

$$\frac{1}{a}\Delta v_\chi = \kappa = \dot{\delta} = cH\delta\,, \qquad c \equiv \dot{D}/HD\,, \tag{53}$$

where $D(t)$ is the time-dependent part of δ. Since $D(t)$ is typically proportional to t^n with n of the order of unity [27], it follows that $c = \mathcal{O}(1)$. Hence we get

$$\left|\frac{(\dot{F}/a)\Delta v_\chi}{2Q^2\delta\rho}\right| \simeq \left|\frac{\dot{\phi}}{QH}\right|\,. \tag{54}$$

As long as the condition,

$$|\dot{\phi}| \ll |QH|\,, \tag{55}$$

is satisfied, we have that $|2Q^2\delta\rho| \gg |(\dot{F}/a)\Delta v_\chi|$ and

$$\left(\frac{M^2}{F} - \frac{\Delta}{a^2}\right)\delta F \simeq 2Q^2\rho_m\delta\,. \tag{56}$$

In the previous section we showed that $|\dot{\phi}|$ is much smaller than H for the potential (4). Hence the condition (55) holds well for the values of $|Q|$ which are not very much smaller than 1.

Equation (56) shows that the perturbation δF is sourced by the matter perturbation δ. Hence Eq. (49) can be written as

$$\dot{\kappa} + 2H\kappa - 4\pi\rho_m G_{\rm eff}\delta = \frac{1}{a^2}\left[(\nabla v_\chi)\cdot(\nabla v_\chi)^{,i}\right]_{,i}\,, \tag{57}$$

where

$$G_{\rm eff}\delta \equiv \frac{1}{8\pi F}\left(\delta - \frac{1}{\rho_m}\frac{\Delta\delta F}{a^2}\right)\,. \tag{58}$$

We introduce an effective potential Φ and a peculiar velocity $\boldsymbol{u}$ as follows:

$$\frac{\Delta\Phi}{a^2} = 4\pi\rho_m G_{\rm eff}\delta\,, \tag{59}$$

$$\boldsymbol{u} = -\nabla v_\chi\,. \tag{60}$$

If one defines an effective gravitational potential $\Psi = \varphi + aHv_\chi$ in Eq. (30), it follows that $\Delta\Psi/a^2 = 4\pi\tilde{G}_{\rm eff}\rho_m\delta$ at linear order, where $\tilde{G}_{\rm eff}$ is different from $G_{\rm eff}$ by the sign of $\Delta F/a^2$. Taking the time-derivative of Eq. (47) together with the use of Eq. (57), we get

$$\frac{\partial^2\delta}{\partial t^2} + 2H\frac{\partial\delta}{\partial t} = \frac{1}{a^2}\nabla\cdot(1+\delta)\nabla\Phi + \frac{1}{a^2}\frac{\partial^2}{\partial x^i\partial x^j}\left(u^i u^j\right)\,. \tag{61}$$

This is our master equation that is used to compute the skewness of matter density perturbations in the next section.

IV. SKEWNESS IN MODIFIED GRAVITY

In this section we study the skewness of matter perturbations for the action (2) with the potential (4). The skewness will be derived analytically in the matter-dominated epoch by using Eq. (61).

A. First-order perturbations

We write the solution to Eq. (61) in the form $\delta = \delta^{(1)} + \delta^{(2)} + \cdots$, where the subscripts represent the orders of perturbations. The equation for the first-order perturbation $\delta^{(1)}$ is

$$\frac{\partial^2\delta^{(1)}}{\partial t^2} + 2H\frac{\partial\delta^{(1)}}{\partial t} - 4\pi\rho_m G_{\rm eff}^{(1)}\,\delta^{(1)} = 0\,. \tag{62}$$

We express the first-order perturbations $\delta^{(1)}$ and $\delta F^{(1)}$ in the plane-wave form: $\delta^{(1)} = \int \delta_k^{(1)}(t)e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\,{\rm d}^3\boldsymbol{k}$ and $\delta F^{(1)} = \int \delta F_k^{(1)}(t)e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\,{\rm d}^3\boldsymbol{k}$. From Eq. (56) we obtain

$$\delta F_k^{(1)}(t) = \frac{2Q^2\rho_m}{M^2/F + k^2/a^2}\delta_k^{(1)}(t)\,. \tag{63}$$

Then the temporal part of Eq. (62) satisfies

$$\ddot{\delta}_k^{(1)}(t) + 2H\dot{\delta}_k^{(1)}(t) - 4\pi\rho_m G_{\rm eff}^{(1)}\delta_k^{(1)}(t) = 0\,, \tag{64}$$

where

$$G_{\rm eff}^{(1)} = \frac{1}{8\pi F}\frac{(k^2/a^2)(1+2Q^2) + M^2/F}{(k^2/a^2) + M^2/F}\,. \tag{65}$$

In the early stage of the matter era, the mass M is sufficiently heavy to satisfy the condition $M^2/F \gg k^2/a^2$. In this regime we have $G_{\rm eff}^{(1)} \simeq 1/8\pi F \simeq G$, thus mimicking the evolution in General Relativity. At late times it happens that the perturbations enter the regime $M^2/F \ll k^2/a^2$. This case corresponds to $G_{\rm eff}^{(1)} \simeq (1+2Q^2)/8\pi F$, thus showing the deviation from General Relativity. The transition from the former regime to the latter regime occurs at a redshift z_k given by [27]:

$$z_k \simeq \left[\left(\frac{k}{a_0 H_0}\frac{1}{Q}\right)^{2(1-p)} \frac{2^p pC}{(1-p)^{1-p}}\frac{1}{(3F_0\Omega_m^{(0)})^{2-p}}\frac{V_0}{H_0^2}\right]^{\frac{1}{4-p}} - 1\,, \tag{66}$$

where a_0 and H_0 are the present values.

Using the derivative with respect to $N = \ln(a)$, Eq. (64) can be written as

$$\delta_k^{(1)''} + \left(\frac{1}{2} - \frac{3}{2}w_{\rm eff}\right)\delta_k^{(1)'} - 12\pi F\Omega_m G_{\rm eff}^{(1)}\delta_k^{(1)} = 0\,, \tag{67}$$

where a prime represents the derivative in terms of N. As we explained in Sec. II, we are considering the case in which the matter era is realized by the point (M2) with $|\lambda| \gg |Q| = \mathcal{O}(1)$. Since $\Omega_m \simeq 1$ and $w_{\rm eff} \simeq 0$ in this case, we get the following solutions

$$\delta_k^{(1)}(t) \propto \begin{cases} t^{2/3}, & \text{for } z \gg z_k, \\ t^{\frac{1}{6}(\sqrt{25+48Q^2}-1)}, & \text{for } z \ll z_k. \end{cases} \tag{68}$$

In these asymptotic regimes the growth rates of first-order perturbations are independent of the wavenumber k. The growth rate is constrained to be $s \equiv \dot{\delta}_k^{(1)}/H\delta_k^{(1)} \lesssim 2$ from observational data, which gives the bound $|Q| \lesssim 1$ [27].

B. Conditions for the validity of approximations to reach Eq. (61)

In order to reach Eq. (47), we have employed the approximation that the field-dependent term on the r.h.s of Eq. (45) is neglected relative to the term $\Delta v_\chi/a$. We have also neglected some of first-order terms in Eq. (48) relative to the second-order term $\frac{1}{a^2}\left[(\nabla v_\chi)\cdot(\nabla v_\chi)^{,i}\right]_{,i}$ in Eq. (49). We derive conditions under which these approximations are justified.

We write the temporal part of the first-order perturbation $v_\chi^{(1)}$ as $(v_\chi)_k^{(1)}(t)$. Using Eqs. (63) and (53) together with the relation $\rho_m \simeq 3FH^2$ that holds during the matter era, we find

$$\left|\frac{H\delta F_k^{(1)}}{F}\right| \simeq \frac{Q^2H^2}{M^2/F + k^2/a^2}\left|H\delta_k^{(1)}\right| \simeq \frac{Q^2H^2}{M^2/F + k^2/a^2}\left|\frac{1}{a}(\Delta v_\chi)_k^{(1)}\right|. \tag{69}$$

As we showed in Sec. II the condition, $M^2/F \gg H^2$, holds for the potential (4). Moreover we are considering sub-horizon modes deep inside the horizon, i.e., $k^2 \gg a^2H^2$. This leads to the relation $|H\delta F_k^{(1)}/F| \ll |(\Delta v_\chi)_k^{(1)}/a|$ in Eq. (69). Since the field ϕ is nearly frozen at instantaneous minima of its effective potential, we have the relation $|\delta\dot{F}_k^{(1)}| \lesssim |H\delta F_k^{(1)}|$ and hence $|\delta\dot{F}_k^{(1)}/F| \ll |(\Delta v_\chi)_k^{(1)}/a|$. The following inequality is also satisfied:

$$\left|\frac{1}{F}\omega\dot{\phi}\delta\phi_k^{(1)}\right| = \left|\frac{(1-6Q^2)\dot{\phi}}{2Q}\frac{\delta F_k^{(1)}}{F}\right| \ll \left|\frac{1}{a}(\Delta v_\chi)_k^{(1)}\right|, \tag{70}$$

where we used Eq. (55). The above discussion shows that the field-dependent terms in Eq. (45) are neglected relative to the term $\Delta v_\chi/a$, thus ensuring the validity of the approximation, $\kappa^{(1)} \simeq \Delta v_\chi^{(1)}/a$, for the modes deep inside the Hubble radius.

The second-order term on the r.h.s. of Eq. (49) is of the order of $H^2\left|\delta_k^{(1)}\right|^2$ by employing the first-order solution (53). Meanwhile one of the first-order term, $H^2\delta F_k^{(1)}/F$, in Eq. (48) has been already estimated in Eq. (69). The former is larger than the latter provided that

$$|\delta_k^{(1)}| \gg \frac{Q^2H^2}{M^2/F + k^2/a^2}. \tag{71}$$

The r.h.s. of Eq. (71) is much smaller than unity because of the condition $\{M^2/F, k^2/a^2\} \gg H^2$. One can show that, under the condition (71), other field-dependent first-order terms on the l.h.s. of Eq. (48) can be negligible relative to the term $\frac{1}{a^2}\left[(\nabla v_\chi)\cdot(\nabla v_\chi)^{,i}\right]_{,i}$. In summary, the master equation (61) we have approximately derived in the previous section is trustable under the conditions (55) and (71).

C. Second-order perturbations and skewness

The second-order perturbation $\delta^{(2)}$ satisfies

$$\frac{\partial^2\delta^{(2)}}{\partial t^2} + 2H\frac{\partial\delta^{(2)}}{\partial t} - 4\pi\rho_m G_{\rm eff}^{(2)}\delta^{(2)} = 4\pi G_{\rm eff}^{(1)}\rho_m\left(\delta^{(1)}\right)^2 + \frac{1}{a^2}\delta_{,i}^{(1)}\Phi_{,i}^{(1)} + \frac{1}{a^2}\left[u^{i(1)}u^{j(1)}\right]_{,ij}, \tag{72}$$

where $G_{\rm eff}^{(2)}\delta^{(2)} = [\delta^{(2)} - \Delta\delta F^{(2)}/(\rho_m a^2)]/(8\pi F)$. When the growth rate of perturbations is dependent on k, the gravitational constant $G_{\rm eff}^{(2)}$ is generally different from $G_{\rm eff}^{(1)}$. In the following we study the regime of the massless limit

($M \to 0$) in which the growth rate of the first-order perturbation is independent of k, i.e, $\delta^{(1)} = D(t)\delta_1(\boldsymbol{x})$ with $D(t) = t^{(\sqrt{25+48Q^2}-1)/6}$. In this regime we have $G_{\rm eff}^{(2)} = G_{\rm eff}^{(1)} = (1+2Q^2)/8\pi F$, so we simply adopt the notation, $G_{\rm eff}$, instead of $G_{\rm eff}^{(1)}$ and $G_{\rm eff}^{(2)}$. The General Relativistic case ($M \to \infty$) is recovered by taking the limit $Q \to 0$.

The first-order solution to $\boldsymbol{u}$ can be obtained by solving Eq. (53), i.e., $\nabla \cdot \boldsymbol{u}^{(1)} = -a\dot{\delta}^{(1)}$. It is given by

$$\boldsymbol{u}^{(1)} = -\frac{a\dot{D}}{4\pi}\int \frac{(\boldsymbol{x}-\boldsymbol{x}')\delta_1(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|^3}\,{\rm d}^3\boldsymbol{x}' = \frac{a\dot{D}}{4\pi}\Delta_{,i}\,, \tag{73}$$

where $\Delta_{,i}$ is a spatial derivative of the quantity:

$$\Delta(\boldsymbol{x}) \equiv \int \frac{\delta_1(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}{\rm d}^3\boldsymbol{x}'\,. \tag{74}$$

This satisfies the relation $\Delta_{,ii} = -4\pi\delta_1(\boldsymbol{x})$.

The last term on the r.h.s. of Eq. (72) yields

$$\begin{aligned}\frac{1}{a^2}\left[u^{i(1)}u^{j(1)}\right]_{,ij} &= \frac{1}{a^2}\left[u^{i(1)}_{,i}u^{j(1)}_{,j} + 2u^{i(1)}_{,ij}u^{j(1)} + u^{i(1)}_{,j}u^{j(1)}_{,i}\right] \\ &= \dot{D}^2\left[\delta_1^2 - \frac{1}{2\pi}\delta_{1,j}\Delta_{,j} + \frac{1}{16\pi^2}\Delta_{,ij}\Delta_{,ij}\right]\,. \end{aligned}\tag{75}$$

We write the solution of Eq. (72) in the form [31]

$$\delta^{(2)} = \delta_a^{(2)} + \delta_b^{(2)}\,, \tag{76}$$

where $\delta_a^{(2)}$ and $\delta_b^{(2)}$ satisfy

$$\frac{\partial^2\delta_a^{(2)}}{\partial t^2} + 2H\frac{\partial\delta_a^{(2)}}{\partial t} - 4\pi G_{\rm eff}\rho_m\delta_a^{(2)} = 4\pi G_{\rm eff}\rho_m D^2\delta_1^2 + \frac{D}{a^2}\Phi^{(1)}_{,i}\delta_{1,i}\,, \tag{77}$$

$$\frac{\partial^2\delta_b^{(2)}}{\partial t^2} + 2H\frac{\partial\delta_b^{(2)}}{\partial t} - 4\pi G_{\rm eff}\rho_m\delta_b^{(2)} = \dot{D}^2\left[\delta_1^2 - \frac{1}{2\pi}\delta_{1,i}\Delta_{,i} + \frac{1}{16\pi^2}\Delta_{,ij}\Delta_{,ij}\right]\,. \tag{78}$$

Since $\Phi^{(1)}_{,i} = -G_{\rm eff}\rho_m a^2 D\Delta_{,i}$ from Eq. (59), the r.h.s. of Eq. (77) is given by $4\pi G_{\rm eff}\rho_m D^2[\delta_1^2 - (1/4\pi)\Delta_{,i}\delta_{1,i}]$. Writing the solution of $\delta_a^{(2)}$ as $\delta_a^{(2)} = E_a(t)\delta_a(\boldsymbol{x})$, we obtain the following equation for the temporal part:

$$\ddot{E}_a + 2H\dot{E}_a - 4\pi G_{\rm eff}\rho_m E_a = 4\pi G_{\rm eff}\rho_m D^2\,, \tag{79}$$

where the spatial part is given by $\delta_a(\boldsymbol{x}) = \delta_1^2 - (1/4\pi)\Delta_{,i}\delta_{1,i}$. Expressing the solution of Eq. (78) in the form $\delta_b^{(2)} = E_b(t)\delta_b(\boldsymbol{x})$, we get

$$\ddot{E}_b + 2H\dot{E}_b - 4\pi G_{\rm eff}\rho_m E_b = \dot{D}^2\,, \tag{80}$$

and $\delta_b(\boldsymbol{x}) = \delta_1^2 - (1/2\pi)\Delta_{,i}\delta_{1,i} + (1/16\pi^2)\Delta_{,ij}\Delta_{,ij}$.

We then find the following solution for second-order perturbations:

$$\begin{aligned}\delta^{(2)}(t,\boldsymbol{x}) &= E_a(t)\left[\delta_1^2 - \frac{1}{4\pi}\Delta_{,i}\delta_{1,i}\right] + E_b(t)\left[\delta_1^2 - \frac{1}{2\pi}\Delta_{,i}\delta_{1,i} + \frac{1}{16\pi^2}\Delta_{,ij}\Delta_{,ij}\right]\,, \\ &= \frac{D^2+E_a}{2}\delta_1^2 - \frac{D^2}{4\pi}\Delta_{,i}\delta_{1,i} + \frac{D^2-E_a}{32\pi^2}\Delta_{,ij}\Delta_{,ij}\,. \end{aligned}\tag{81}$$

In the second line we employed the fact that E_a and E_b are related each other via the relation $E_a + 2E_b = D^2$.

We assume that the initial distribution of perturbations is Gaussian so that it is described by an auto-correlation function $\xi(\boldsymbol{x})$ satisfying

$$\begin{aligned}&\langle\delta(\boldsymbol{x})\rangle = 0\,, \quad \langle\delta(\boldsymbol{x}_1)\delta(\boldsymbol{x}_2)\rangle = \xi(|\boldsymbol{x}_1-\boldsymbol{x}_2|)\,, \quad \langle\delta(\boldsymbol{x}_1)\delta(\boldsymbol{x}_2)\delta(\boldsymbol{x}_3)\rangle = 0\,, \\ &\langle\delta(\boldsymbol{x}_1)\delta(\boldsymbol{x}_2)\delta(\boldsymbol{x}_3)\delta(\boldsymbol{x}_4)\rangle = \xi(|\boldsymbol{x}_1-\boldsymbol{x}_2|)\xi(|\boldsymbol{x}_3-\boldsymbol{x}_4|) + \xi(|\boldsymbol{x}_1-\boldsymbol{x}_3|)\xi(|\boldsymbol{x}_2-\boldsymbol{x}_4|) + \xi(|\boldsymbol{x}_1-\boldsymbol{x}_4|)\xi(|\boldsymbol{x}_2-\boldsymbol{x}_3|)\,. \end{aligned}\tag{82}$$

Since $\langle(\delta^{(1)})^3\rangle = 0$, the quantity $\langle\delta^3\rangle$ is given by $\langle\delta^3\rangle = 3\langle(\delta^{(1)})^2\delta^{(2)}\rangle$ to the lowest order. We then have

$$\langle\delta^3\rangle = \frac{3}{2}D^2(D^2+E_a)\langle\delta_1^4\rangle - \frac{3}{4\pi}D^4\langle\delta_1^2\delta_{1,i}\Delta_{,i}\rangle + \frac{3}{32\pi^2}D^2(D^2-E_a)\langle\delta_1^2\Delta_{,ij}\Delta_{,ij}\rangle\,. \tag{83}$$

Since the each ensemble average in Eq. (83) satisfies the relations $\langle\delta_1^4\rangle = 3\xi(0)^2$, $\langle\delta_1^2\delta_{1,i}\Delta_{,i}\rangle = 4\pi\xi(0)^2$ and $\langle\delta_1^2\Delta_{,ij}\Delta_{,ij}\rangle = \frac{80\pi^2}{3}\xi(0)^2$ [30], we obtain the skewness

$$S_3 \equiv \frac{\langle\delta^3\rangle}{\langle\delta^2\rangle^2} = 4 + 2\frac{E_a}{D^2}\,, \tag{84}$$

where we used $\langle\delta_1^2\rangle = \xi(0)^2$. Hence the skewness is determined by the second-order growth rate E_a relative to the squared of the first-order growth rate D.

Equation (79) can be written as

$$E_a'' + \left(\frac{1}{2} - \frac{3}{2}w_{\rm eff}\right)E_a' - 12\pi F G_{\rm eff}\Omega_m E_a = 12\pi F G_{\rm eff}\Omega_m D^2\,. \tag{85}$$

Recall that $G_{\rm eff} = (1+2Q^2)/8\pi F$ in the limit $M \to 0$. During the matter era realized by the point (M2) we have $\Omega_m \simeq 0$ and $w_{\rm eff} \simeq 0$, in which case Eq. (85) reduces [1]

$$E_a'' + \frac{1}{2}E_a' - \frac{3}{2}(1+2Q^2)E_a = \frac{3}{2}(1+2Q^2)D^2\,. \tag{86}$$

Using the first-order solution $D = e^{\frac{1}{4}(\sqrt{25+48Q^2}-1)N}$, we get the following special solution for Eq. (86):

$$E_a = \frac{6(1+2Q^2)}{19+36Q^2-\sqrt{25+48Q^2}}e^{\frac{1}{2}(\sqrt{25+48Q^2}-1)N}\,. \tag{87}$$

Hence the skewness in the regime of the massless limit is given by

$$S_3 = \frac{4[22+42Q^2-\sqrt{25+48Q^2}]}{19+36Q^2-\sqrt{25+48Q^2}}\,. \tag{88}$$

The General Relativistic case is recovered by taking the limit $Q \to 0$:

$$S_3 = 34/7\,. \tag{89}$$

This agrees with the skewness in the Einstein-de Sitter Universe [30] (pressureless matter without cosmological constant).

In Fig. 1 we plot the analytic value (88) as a function of $|Q|$. The skewness shows some difference compared to the Einstein-de Sitter value 34/7 for $|Q| > 0.1$. When $|Q| = 1$ we have $S_3 = 4.775$, which is different from the value 34/7 only by 1.7 %. For the potential (4) the first-order perturbation $\delta_k^{(1)}$ evolves from the regime $M^2/F \gg k^2/a^2$ to the regime $M^2/F \ll k^2/a^2$ for the modes relevant to large-scale structure. Hence the skewness tends to evolves from the value 34/7 to the asymptotic value given in Eq. (88). The estimation (88) has been derived by neglecting the transient phase around the redshift z_k. Since this transition occurs quickly for the models that satisfy local gravity constraints ($p > 0.7$) [8, 20, 27], it is unlikely that the skewness is altered significantly by the presence of this transient phase.

The estimation (88) does not take into account the evolution in the late-time accelerated epoch. In the ΛCDM model the numerical analysis shows that the skewness increases a bit during the accelerated phase from the value 34/7 (= 4.857) to the present value 4.865 (at $\Omega_m = 0.28$). This corresponds to the growth of 0.16 % only. We have checked that this situation does not change much even in the presence of the coupling Q. Hence the difference of the present values of the skewness from that in the ΛCDM model is only less than a few percent. This shows that the skewness provides a robust prediction for the picture of gravitational instability from Gaussian initial conditions, including scalar-tensor models with large couplings ($|Q| \lesssim 1$).

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[1] Note that the skewness was calculated in Ref. [39] when the matter era is realized by the point (M1). As we already mentioned, the point (M1) can not be used for the matter era when the coupling $|Q|$ is of the order of unity.

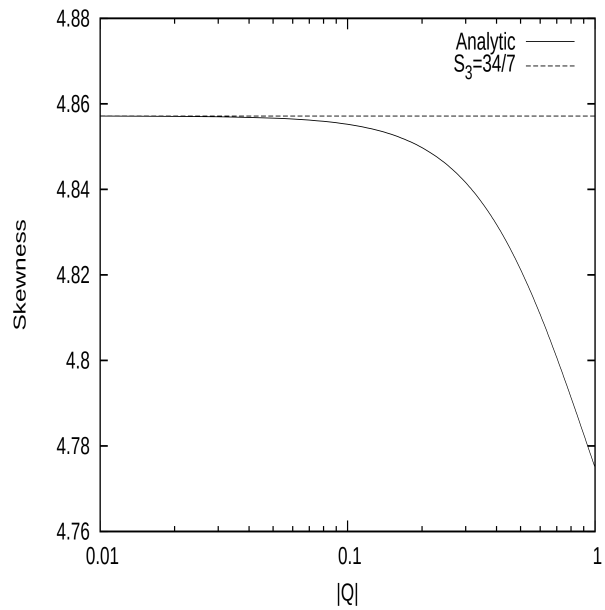

FIG. 1: The analytic estimation (88) of the skewness during the matter-dominated epoch in the regime of the massless limit ($M \to 0$). With the increase of $|Q|$ the skewness gets smaller compared to the value $S_3 = 34/7$ in the Einstein-de Sitter Universe. However the difference from the Einstein-de Sitter case is only less than 1.7 % for $|Q| \leq 1$.

V. CONCLUSIONS

In this paper we have studied the evolution of second-order matter density perturbations in a class of modified gravity models that satisfy local gravity constraints. We have considered the scalar-tensor action (2), which is equivalent to Brans-Dicke action (1) with the correspondence $1/(2Q^2) = 3 + 2\omega_{\rm BD}$. In the presence of a field potential it is possible to satisfy local gravity constraints (LGC) even when $|Q|$ is of the order of unity. In fact the potential (4) is designed to have a large mass in the region of high density for the consistency with LGC. This covers the models proposed by Hu and Sawicki [16] and Starobinsky [8] in the context of $f(R)$ gravity ($Q = -1/\sqrt{6}$).

Starting from second-order relativistic equations of cosmological perturbations, we have derived the equation (61) of matter density fluctuations approximately. In so doing we employed the approximation that first-order perturbations in the scalar field ϕ is neglected relative to second-order matter and velocity perturbations. This is valid under the conditions (55) and (71), both of which can be naturally satisfied for the values of Q we are interested in ($0.1 \lesssim |Q| \lesssim 1$). Compared to the ΛCDM model, the effective gravitational constant $G_{\rm eff}$ is subject to change at the late epoch of the matter era. This leads to the larger growth rate of first-order matter perturbations ($\delta_k^{(1)} \propto t^{(\sqrt{25+48Q^2}-1)/6}$) compared to the standard case ($\delta_k^{(1)} \propto t^{2/3}$).

The skewness of matter distributions is determined by the second-order growth factor E_a relative to the squared of the first-order growth factor D. In the "scalar-tensor regime" where the effective gravitational constant is given by $G_{\rm eff} \simeq (1+2Q^2)/8\pi F$, we have derived the analytic expression (88) of the skewness in the matter-dominated epoch. In the "General Relativistic regime" where $G_{\rm eff} \simeq 1/8\pi F \simeq G$, we have reproduced the standard value $S_3 = 34/7$ in the Einstein-de Sitter Universe. In modified gravity models with $|Q| \lesssim 1$, the analytic value (88) of the skewness in the asymptotic regime of the matter era is different from the value 34/7 only less than a few percent. Even if we take into account the evolution of perturbations during the accelerated phase, the difference of the skewness relative to the ΛCDM model remains to be small. The above result comes from the fact that the ratio of the second-order growth rate relative to the first-order one has a weak dependence on the coupling Q.

When $|Q| = \mathcal{O}(1)$ the growth rate of first-order matter perturbations is significantly different from that in the ΛCDM model. This gives rise to large modifications to the matter power spectrum as well as to the convergence spectrum in weak lensing, while the skewness is hardly distinguishable from that in ΛCDM model. This fact can be useful to discriminate large coupling scalar-tensor models among many other dark energy models from future high-precision observations.

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Appendix

In this Appendix, we estimate the order of second-order terms that we have neglected in Eqs. (50) and (51). The dominant contribution of such second-order terms comes from the third-derivative of the potential, i.e., $V_{,\phi\phi\phi}\delta\phi^2$ [44]. Compared to the field-mass dependent term $V_{,\phi\phi}\delta\phi$ on the l.h.s. of Eq. (50), we have $(V_{,\phi\phi\phi}\delta\phi^2)/(V_{,\phi\phi}\delta\phi) \approx -\delta\phi/\phi$ for the potential (4) under the condition $|Q\phi| \ll 1$.

Since the field stays around the instantaneous minimum given in Eq. (18), the field ϕ can be estimated as

$$\phi \simeq 3(1-p)Q\frac{H^2}{M^2}\,, \tag{90}$$

where we used Eq. (21). In deriving this, we have also employed the approximate relation $\rho_m \approx 3H^2$. Note that the order of ρ_m is not different from $3H^2$ even at the present epoch. Meanwhile, from Eq. (63), the first-order perturbation $\delta\phi_k^{(1)}$ in the Fourier space during the matter era is given by

$$\delta\phi_k^{(1)} \simeq -\frac{3QH^2}{M^2+k^2/a^2}\delta_k^{(1)}\,. \tag{91}$$

Hence we obtain the ratio

$$\frac{\delta\phi_k^{(1)}}{\phi} \simeq -\frac{1}{1-p}\frac{M^2}{M^2+k^2/a^2}\delta_k^{(1)}\,, \tag{92}$$

which shows that $|\delta\phi_k^{(1)}/\phi| \ll 1$ for $\delta_k^{(1)} \ll 1$.

The presence of the second-order term $V_{,\phi\phi\phi}\delta\phi^2$ gives rise to a correction of the order $M^2\delta\phi_k^{(1)}/\phi$ to the mass squared M^2 in Eq. (65). In two asymptotic regimes (i) $M^2 \gg k^2/a^2$ and (ii) $M^2 \ll k^2/a^2$, this appears only as next-order corrections to the small expansion parameters $(k^2/a^2)/M^2$ [regime (i)] and $M^2/(k^2/a^2)$ [regime (ii)].

In the regime $M^2 \gg k^2/a^2$ the correction from the term $V_{,\phi\phi\phi}\delta\phi^2$ to the effective gravitational constant $G_{\rm eff}^{(1)}$ is estimated as $\delta G_{\rm eff}^{(1)} \approx Q^2(k^2/a^2M^2)\delta\phi_k^{(1)}/\phi$. This gives the correction to the third term on the l.h.s. of Eq. (57) in the Fourier space:

$$\left|4\pi\rho_m\,\delta G_{\rm eff}^{(1)}\,\delta_k^{(1)}\right| \approx \frac{Q^2}{1-p}\frac{k^2/a^2}{M^2}H^2{\delta_k^{(1)}}^2\,, \tag{93}$$

which is much smaller than the second-order term on the r.h.s. of Eq. (57) that is of the order of $H^2{\delta_k^{(1)}}^2$.

In the regime $M^2 \ll k^2/a^2$ we have $\delta G_{\rm eff}^{(1)} \approx Q^2M^2(a^2/k^2)\delta\phi_k^{(1)}/\phi$ and hence

$$\left|4\pi\rho_m\,\delta G_{\rm eff}^{(1)}\,\delta_k^{(1)}\right| \approx \frac{Q^2}{1-p}\left(\frac{M^2}{k^2/a^2}\right)^2 H^2{\delta_k^{(1)}}^2\,, \tag{94}$$

which is again much smaller than the r.h.s. of Eq. (57).

The above estimation shows that neglecting second-order terms in Eqs. (50) and (51) is justified.

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